Full Waveform Inversion Using Free-Surface Related Multiples as Natural Blended Sources

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SUMMARY

To improve the computational speed of conventional full waveform inversion (FWI), we propose a new FWI method which uses free-surface related multiples as natural blended sources and name it as NBFWI. The proposed method contains four key steps: 1. at selected source locations, forward propagate synthetic wavelet to generate calculated shot records containing both primaries and free-surface related multiples; 2. Compared with recorded field data to obtain waveform residuals; 3. backward propagate waveform residuals and forward propagate recorded field data to produce gradient; 4. use steep descent method or other optimization methods for velocity updating. Compared with other blended sources methods, the NBFWI method has no need of implementing phase encoding algorithms and the free-surface related multiples act as natural blended sources. A Marmousi velocity model is used for numerical tests. The numerical results show how accurate, efficient and stable the proposed approach is when using different number of shot gathers for inversion. Because of the utilization of wide coverage characteristics of free-surface related multiples, the inversion procedure has good convergence even using very few shots. The proposed approach is easy to implement and maybe significant for both fast velocity model building and inversion of sparse acquisition data.

Introduction

In geophysical prospecting, full waveform inversion (FWI) is an attractive method as it can reconstruct a high-resolution velocity model for seismic imaging. Compared to travel-time tomography, it employs no high frequency approximation and takes full advantages of the phase and amplitude information in the seismograms. Many researchers has studied FWI in time spatial domain (Tarantola, 1986) and in frequency domain (Virieux and Operto, 2009), and verified this technology can provide a more accurate estimation of subsurface velocity. However, FWI has some intrinsic drawbacks, such as the high non-linearity of its misfit function with respect to the velocity model and the requirement of sufficient low-frequency data component. In addition, the computational cost of FWI is expensive because it needs to calculate a large number of shots to provide enough resolution.

In order to reduce the computing cost, many researchers have used phase encoding technique for FWI (Krebs et al., 2009; Ben-Hadj-Ali, Operto and Virieux, 2011; Huang and Schuster, 2013). The phase encoding is used to reduce the number of shots by summing different single shot using different rules. After the summation, some super shots are obtained, by inverting these super-shots, the computing time will be decreased, but extra cross-talks may be introduced, so the design of phase encoding rules keeps challenging.

We propose a natural blended sources full waveform inversion (NBFWI) method, which regards the recorded data as natural blended sources. In proposed approach, the gradient is calculated by the cross-correlation between forward propagated recorded data (contains both primaries and free-surface related multiples) and backward propagated data residuals (contains both primary and free-surface related multiple residuals). The other parts of proposed inversion workflow is same as conventional FWI. We test our approach using a Marmousi velocity model, and use 96 shots, 24 shots and 6 shots for velocity inversion. The numerical results show that even using few shots, the inversion result still contains desirable resolution.

Method

The proposed NBFWI method inverts the velocity model through minimizing the least-squares objective function which is defined as following:

$$E = \frac{1}{2}\sum_{\mathrm{s}}\sum_{\mathrm{r}}[\delta(P+M)_{\mathrm{rs}}]^2 , \tag{1}$$

where the primary and free-surface related multiple residual $\delta(P+M)$ is defined as following:

$$\delta(P+M) = P(\mathrm{r}\,|\,\mathrm{s})_{obs} + M(\mathrm{r}\,|\,\mathrm{s})_{obs} - P(\mathrm{r}\,|\,\mathrm{s})_{cal} - M(\mathrm{r}\,|\,\mathrm{s})_{cal} , \tag{2}$$

here $P(\mathrm{r}\,|\,\mathrm{s})_{obs}$ and $P(\mathrm{r}\,|\,\mathrm{s})_{cal}$ denote the observed and calculated primaries respectively. **r** is receiver location and **s** is source location. $M(\mathrm{r}\,|\,\mathrm{s})_{obs}$ and $M(\mathrm{r}\,|\,\mathrm{s})_{cal}$ denote the observed and calculated free-surface related multiples respectively. In our computation process, the primaries and multiples are calculated simultaneously by using a free-surface boundary condition and acoustic finite difference modeling.

In proposed approach, the velocity gradient is computed by the zero-lag cross-correlation of the forward propagated recorded data and backward propagated data residuals, which can be expressed as following:

$$\gamma_k(\mathrm{x}) = \frac{1}{c^3(\mathrm{x})}\sum_{\mathrm{s}}\sum_{\mathrm{r}}\sum_{\mathrm{r}'}\dot{p}_f(\mathrm{x}\,|\,\mathrm{r}')\dot{p}_b(\mathrm{x}\,|\,\mathrm{r}) , \tag{3}$$

where $\dot{p}$ denotes the time derivative of p, $c(\mathbf{x})$ represents the velocity model. In frequency domain, the forward propagated wave-field p_f and the backward propagated wave-field p_b can be expressed as following:

$$p_f(\mathrm{x}\,|\,\mathrm{r}') = (P+M)_{\mathrm{r's}}G(\mathrm{x}\,|\,\mathrm{r}') \tag{4}$$

$$p_b(\mathrm{x}\,|\,\mathrm{r}) = \delta(P+M)_{\mathrm{rs}} G(\mathrm{x}\,|\,\mathrm{r})^* \tag{5}$$

where $G(\mathrm{x}\,|\,\mathrm{r}')$ and $G(\mathrm{x}\,|\,\mathrm{r})$ are Green's functions associated with acoustic wave-equation for the background velocity model, and the symbol $*$ represents complex conjugate.

The other parts of inversion workflow is same as conventional FWI (Tarantola, 1987; Mora, 1987; Zhou et al., 1995). For simplicity, we use a steepest descent method to illustrate the velocity updating procedure as following:

$$c_{k+1}(\mathrm{x}) = c_k(\mathrm{x}) + \alpha_k \cdot \gamma_k(\mathrm{x}), \tag{6}$$

where α_k is the step length, $\gamma_k(\mathrm{x})$ is the gradient for all image points **x** and *k* is the iteration number.

The correct velocity gradient is generated by two kinds of cross-correlation: 1. The cross-correlation between the forward propagated primaries and backward propagated 1^{st}-order free-surface related multiple residuals; 2. The cross-correlation between the forward propagated n^{th}-order free-surface related multiples and backward propagated $(n+1)^{th}$-order free-surface related multiple residuals.

The errors in the velocity gradient are introduced by four types of cross-correlation (Wang et al., 2014): 1. The cross-correlation between the forward propagated primaries and backward propagated m^{th}-order free-surface related multiple residuals (m>=2); 2. The cross-correlation between the forward propagated n^{th}-order free-surface related multiples and backward propagated m^{th}-order free-surface related multiple residuals (m>=n+2); 3. The cross-correlation between the forward propagated primaries and backward propagated primary residuals; 4. The cross-correlation between the forward propagated n^{th}-order free-surface related multiples and backward propagated m^{th}-order free-surface related multiple residuals (n>=m).

During the iteration procedure, the errors in velocity gradient will be added to velocity inversion result and will result in false waveform residuals, but it does not matter, because after the calculation of new gradient, the errors in previous velocity inversion result will be decreased, and its impact on the final inversion result is negligible. We do the velocity inversion iteratively until the updated velocity model satisfies the stopping criterion. Compared with conventional FWI, the proposed method use natural areal sources for inversion, because the free-surface related multiples have wide coverage of the subsurface, so it is possible to use only few shots to get an acceptable velocity inversion result.

Numerical Tests

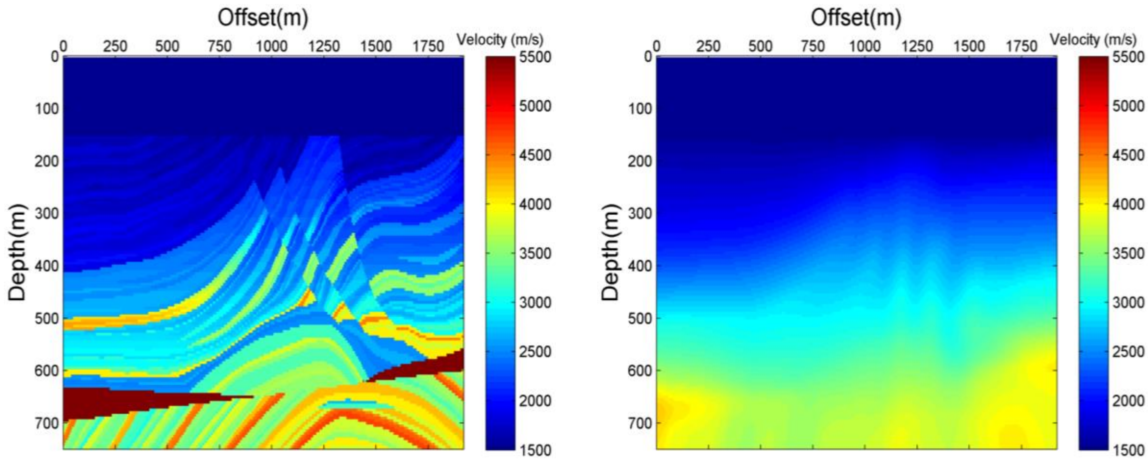
We test our method using a Marmousi velocity model with a water layer on top. Figure 1(a) shows the true model, and Figure 1(b) shows the initial model, a highly smoothed version of the true velocity model. The model size is 384x150 with a grid spacing of 5m. An acoustic finite-difference method is used to generate the synthetic data with a sample interval of 0.5ms. The wavelet is a Ricker wavelet with a 20 Hz peak frequency. The data contains both primaries and multiples.

In the first test, 96 sources and 384 geophones are evenly located along the surface with source and geophone intervals of 20m and 5m, respectively. Figure 2(a) is the inversion result after 18 iterations. It shows that the proposed velocity inversion approach is an effective way to reconstruct velocity model. Figure 2(b) shows how the objective function, primary and free-surface related multiple waveform residual, evolves over the 18 iterations. To show the merits of the approach, the number of shots is reduced. Figure 3(a) and Figure 4(a) are the inverted velocity models of 24 shots and 6 shots respectively. How the corresponding objective functions evolve are shown in Figure 3(b) and Figure 4(b) respectively.

Apparently with more shots, the resolution is improved and the remaining normalized waveform residual is decreased. But even for the inverted model for 6 shots, some details such as the fault and the thin layer are still visible. This means our approach has the potential to offer reconstructed model using less shots, which is practical and economic.

Conclusions

We propose a natural blended sources waveform inversion approach, which directly use complete recorded data (contains both primary and free-surface related multiples) as areal sources. This method utilizes the wide coverage characteristic of free-surface related multiples and can obtain acceptable inversion results using only few shots. The proposed approach potentially can be applied to data with sparse acquisition geometry, e.g. 3D cases or can be used for fast velocity model building.



(a) **(b)**

Figure 1 *(a) is Marmousi velocity model added with a water layer on top and (b) is the initial velocity model, which is a smoothed version of (a).*

Acknowledgements

The research was funded by National Science and Technology Major Project of China (Grant No. 2011ZX05030-004-001).

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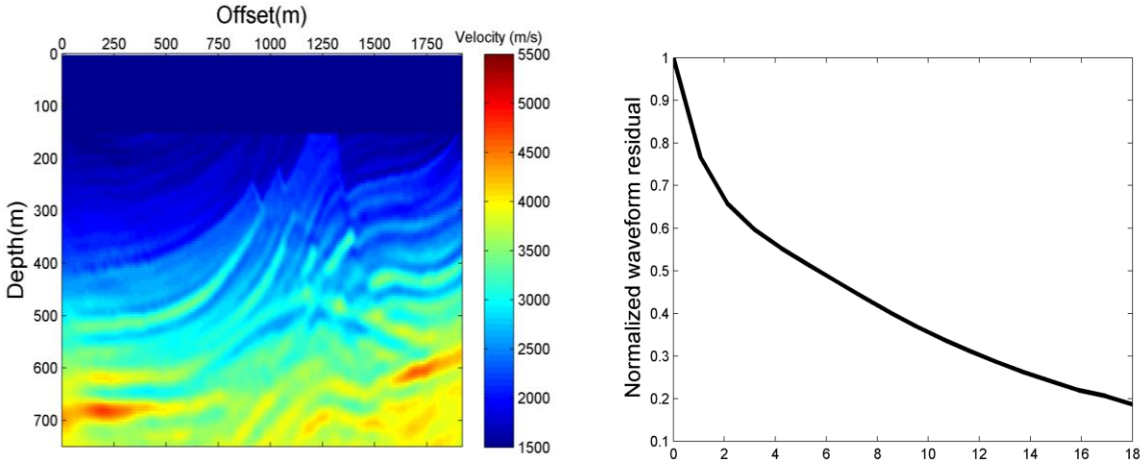
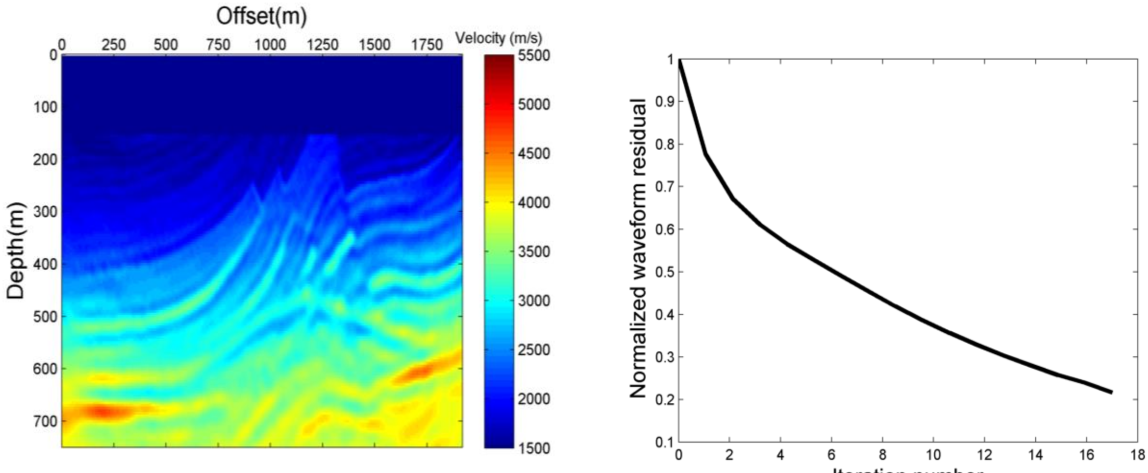


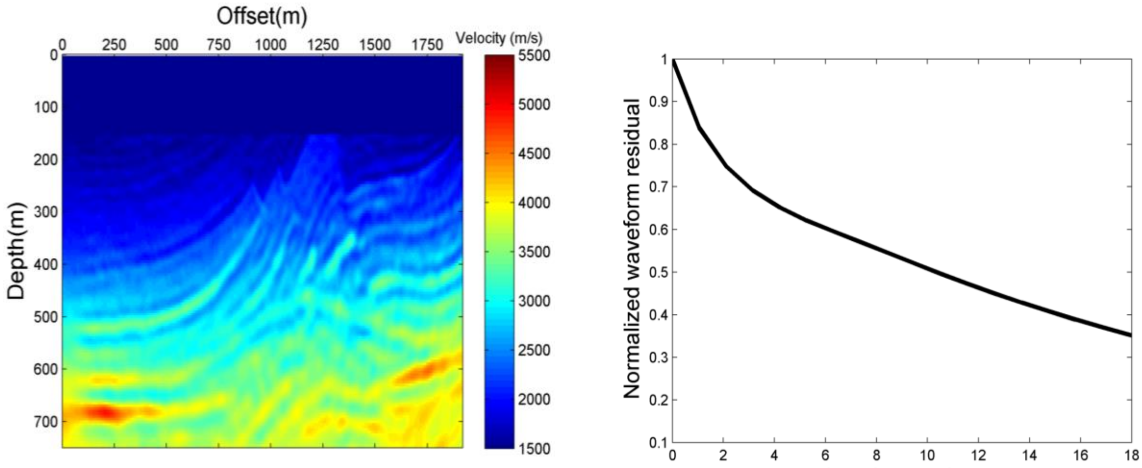
(a) **(b)**

Figure 2 *(a) is the NBFWI result using 96 shots and (b) is the normalized waveform residual versus iteration number curve.*



(a) **(b)**

Figure 3 *(a) is the NBFWI result using 24 shots and (b) is the normalized waveform residual versus iteration number curve.*



(a) ***(a)***

Figure 4 *(a) is the NBFWI result using 6 shots and (b) is the normalized waveform residual versus iteration number curve.*