Seismic Inversion

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About the Author

Gerard Schuster is currently a professor of geophysics at King Abdullah University Science and Technology (KAUST) and an adjunct professor at University of Utah and University of Wyoming. He was the founder and director of the Utah Tomography and Modeling/Migration consortium from 1987 to 2009 and is now the co-director and founder of the Center for Fluid Modeling and Seismic Imaging at KAUST. Schuster helped pioneer seismic interferometry and its practical applications in applied geophysics through his active research program and through his extensive publications. He also has extensive experience in developing innovative migration and inversion methods for both exploration and earthquake seismology.

Schuster has an MS (1982) and a PhD (1984) from Columbia University and was a postdoctoral researcher there from 1984–1985. From 1985 to 2009, he was a professor of geophysics at University of Utah. He left Utah to start his current position as professor of geophysics at KAUST in 2009. He received a number of teaching and research awards while at University of Utah. He was editor of *Geophysics* 2004–2005 and was awarded SEG’s Virgil Kauffman Gold Medal in 2010 for his work in seismic interferometry. He was the SEG Distinguished Lecturer in 2013.
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Preface

This book describes the theory and practice of inverting seismic data for the subsurface rock properties of the earth. The primary application is for inverting reflection and/or transmission data from engineering or exploration surveys, but the methods described also can be used for earthquake studies. I have written this book with the hope that it will be largely comprehensible to scientists and advanced students in engineering, earth sciences, and physics. It is desirable that the reader has some familiarity with certain aspects of numerical computation, such as finite-difference solutions to partial differential equations, numerical linear algebra, and the basic physics of wave propagation (e.g., Snell’s law and ray tracing). For those not familiar with the terminology and methods of seismic exploration, a brief introduction is provided in the Appendix of Chapter 1. Computational labs are provided for most of the chapters, and some field data labs are given as well.

MATLAB and Fortran labs at the end of some chapters are used to deepen the reader’s understanding of the concepts and their implementation. Such exercises are introduced early and geophysical applications are presented in every chapter. For the non-geophysicist, geophysical concepts are introduced with intuitive arguments, and their description by rigorous theory is deferred to later chapters.

The lab exercises in the Computational Toolkit can be found at http://csim.kaust.edu.sa/web/SeismicInversion and http://utam.gg.utah.edu/SeismicInversion/; the exercises can be accessed using the login Paulina and the password Brozina.
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Notation Convention

- \( \mathbb{R}^N \) denotes the \( N \)-dimensional real vector space.
- \( \mathbb{C}^N \) denotes the \( N \)-dimensional complex vector space.
- A column vector will be denoted by boldface lower-case letters. For example, \( \mathbf{x} = [x_1, x_2, \ldots, x_N]^T \) represents the \( N \times 1 \) vector where \( x_i \) is the \( i^{th} \) element of \( \mathbf{x} \).
- A matrix will be denoted by boldface upper-case letters. For example, \( \mathbf{A} \in \mathbb{R}^{M \times N} \) represents an \( M \times N \) real matrix whose \( i^{th} j^{th} \) element is denoted by \( A_{ij} \).
- An order-of-magnitude estimate of a variable whose precise value is unknown is an estimate rounded to the nearest power of ten.
- A scalar will be denoted by lower-case letters.
- Subscripts are usually used to denote the element index of a vector or matrix.
- Superscripts with parentheses are used to denote an iterate of a vector or matrix. For example, \( \mathbf{x}^{(k)} \) denotes the \( k^{th} \) iterate of an iterative scheme.
- \( \mathbf{x} \cdot \mathbf{y} = \mathbf{x}^T \mathbf{y} = \langle \mathbf{x}, \mathbf{y} \rangle = \sum_{i=1}^{N} x_i^* y_i \) represents a dot product or an inner product between finite-dimensional vectors \( \mathbf{x} \) and \( \mathbf{y} \).
- MATLAB syntax is sometimes used to represent vectors or matrices. For example, \( [a \ b; c \ d] \) denotes the matrix
  \[
  \begin{bmatrix}
  a & b \\
  c & d
  \end{bmatrix}
  \]
- \( ||\mathbf{x}||_1 \) denotes the 1-norm of the \( N \times 1 \) vector \( \mathbf{x} \) which is equal to
  \[
  ||\mathbf{x}||_1 = \sum_{i=1}^{N} |x_i|.
  \]
- \( |\mathbf{x}| = ||\mathbf{x}||_2 \) denotes the 2-norm or Euclidean norm of the \( N \times 1 \) vector \( \mathbf{x} \) which is equal to
  \[
  ||\mathbf{x}||_2 = \sqrt{\sum_{i=1}^{N} x_i^2}.
  \]

If the subscript is missing then the 2-norm is indicated, \( l_2 \) for a discrete vector and \( L_2 \) for a well-behaved function of a continuous variable.

- The length of a vector \( \mathbf{x} \) will often be denoted as \( |\mathbf{x}| \) rather than \( ||\mathbf{x}||_2 \).
- \( ||\mathbf{x}||_p \) denotes the \( p \)-norm or Euclidean norm of the \( N \times 1 \) vector \( \mathbf{x} \) which is equal to
  \[
  ||\mathbf{x}||_p = \left( \sum_{i=1}^{N} x_i^p \right)^{\frac{1}{p}}.
  \]
- \( \hat{\mathbf{x}} = \frac{\mathbf{x}}{|\mathbf{x}|} \) denotes the unit vector.
- \( \mathbf{A}^* \) denotes the complex conjugate of the matrix \( \mathbf{A} \).
- \( \mathbf{A}^T \) denotes the transpose of matrix \( \mathbf{A} \). We will often insist it also means the transpose and complex conjugated matrix \( \mathbf{A} \).
- \( \mathbf{A}^\dagger \) denotes the conjugated and transposed matrix \( \mathbf{A} \).
- \( \star \) denotes temporal convolution. For example, assuming \( f(t) \) and \( g(t) \) are real continuous functions of the scalar variable \( t \) and are square integrable then
  \[
  f(t) \star g(t) = \int_{-\infty}^{\infty} f(t - \tau) g(\tau) d\tau = \int_{-\infty}^{\infty} f(\tau) g(t + \tau) d\tau.
  \]  
  (1)
- \( \otimes \) denotes temporal correlation. For example, assuming \( f(t) \) and \( g(t) \) are real continuous functions of the scalar variable \( t \) and are square integrable then
  \[
  f(t) \otimes g(t) = f(-t) \star g(t) = \int_{-\infty}^{\infty} f(\tau) g(t + \tau) d\tau.
  \]  
  (2)
- \( \mathcal{F}[f(t)] = F(\omega) \) denotes the Fourier transform of \( f(t) \) and \( \mathcal{F}^{-1}[F(\omega)] \) is the inverse Fourier transform of \( F(\omega) \). For example,
  \[
  F(\omega) = \mathcal{F}[f(t)] = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt,
  \]
  \[
  f(t) = \mathcal{F}^{-1}[F(\omega)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{i\omega t} d\omega,
  \]
  \[
  \frac{d^n f(t)}{dt^n} = \frac{1}{2\pi} \int_{-\infty}^{\infty} (i\omega)^n F(\omega) e^{i\omega t} d\omega,
  \]
  \[
  f(-t) = \mathcal{F}^{-1}[F(\omega)^*],
  \]
  \[
  \mathcal{F}[f(t) \star g(t)] = F(\omega) G(\omega),
  \]
\[ F[f(t) \otimes g(t)] = F[f(-t) \star g(t)] = F(\omega)^* G(\omega), \]

\[ f(t) \star g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) G(\omega) e^{i\omega t} d\omega, \]

\[ f(t) \otimes g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)^* G(\omega) e^{i\omega t} d\omega, \]

\[ f(t) \otimes g(t)|_{t=0} = \int_{-\infty}^{\infty} f(\tau) g(\tau) d\tau \]
\[ = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)^* G(\omega) d\omega, \]

\[ f(t) \otimes f(t)|_{t=0} = \int_{-\infty}^{\infty} f(\tau)^2 d\tau \]
\[ = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(\omega)|^2 d\omega. \]  

(3)

- The Dirac delta function \( \delta(t) \) is a generalized function (Zemanian, 1965) that is zero everywhere on the real line, except at \( t = 0 \). The Dirac delta function has a broadband spectrum with the constant amplitude 1:

\[ \delta(t-t') = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\omega(t-t')} d\omega. \]

(4)

For a smooth function \( f(\tau) \), the delta function has the sifting property:

\[ f(t) = \int_{-\infty}^{\infty} f(\tau) \delta(\tau-t) d\tau. \]

(5)
Abbreviations

ABC  absorbing boundary condition  KM  Kirchhoff migration
ADCIG  angle-domain common image gather  LSM  least squares migration
CAG  common angle gather  LSRTM  least squares reverse time migration
CFL  Courant-Friedrichs-Lewy  MD  migration deconvolution
CG  conjugate gradient  MVA  migration velocity analysis
CIG  common image gather  NLCG  nonlinear conjugate gradient
CMG  common midpoint gather  NMO  normal moveout
COG  common offset gather  PDE  partial differential equation
CSG  common shot gather  PML  perfectly matched layer
DFP  Davidon-Fletcher-Powell  PSTM  prestack time migration
DM  diffraction-stack migration  QN  quasi-Newton
DOD  domain of dependence  RTM  reverse time migration
DSO  differential semblance optimization  SD  steepest descent
EWT  early arrival wave equation tomography  SE  spectral element
FD  finite difference  SLS  standard linear solid
FE  finite element  SPD  symmetric positive definite
FWI  full waveform inversion  SSP  surface seismic profile
GCV  generalized cross validation  SV  singular vector
GDM  generalized diffraction-stack migration  VSP  vertical seismic profile
GDSO  generalized differential semblance optimization  WT  wave equation traveltime tomography
GIDI  generalized image domain inversion  WTW  wave equation traveltime and waveform tomography
GOM  Gulf of Mexico  ZO  zero offset
IDI  image domain inversion
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