Least-squares Reverse-time Migration with Statistical Sampling

W. Dai, K. Jiao, D. Coles & R.T. Coates

SUMMARY

Least-squares reverse-time migration (LSRTM) is a recently developed imaging algorithm, where the image is produced using an iterative inversion process. Tests on synthetic and real data have shown the promise that LSRTM can improve the image quality by balancing the reflector amplitudes, suppressing migration artifacts and enhancing the image resolution. However, each iteration of the process is comparable in computational cost to a conventional RTM and typically 10 to 20 iterations are required to converge; thus the application of LSRTM has been limited. In this paper we incorporate statistical sampling with LSRTM to reduce its computational cost. The empirical results suggest that this approach reduces the cost of LSRTM to 2 or 3 times that of conventional RTM while retaining most of the quality improvements.

Introduction

Least-squares reverse time migration (LSRTM) has been shown to be able to improve image quality over the conventional RTM method by balancing the reflector amplitudes and improving the image resolution (Dai and Schuster 2013; Dai et al. 2013; Zhang et al. 2013). During the linear inversion process, the LSRTM takes into account the Hessian matrix, which is usually approximated to be diagonal in conventional migration, and to improve the image quality over iterations. One drawback of LSRTM is that its computational cost is usually much higher than the conventional method.

Many authors (such as Schuster et al. 2011) have attempted to address the high computational cost by shot-gather encoding: phase encoding (Romero et al. 2000), polarity encoding (Krebs et al. 2009), time-dithering (Dai et al. 2012), random shot-location encoding (Boonyasiriwat and Schuster 2010), plane-wave encoding (Vigh and Starr 2008; Dai and Schuster 2013) and frequency division (Dai et al. 2013). These techniques have also been applied previously to full waveform inversion (FWI) and have achieved limited success.

The concept of statistical sampling was introduced to FWI by van Leeuwen and Herrmann (2012), where a randomly chosen subset of the data (shots) can give an adequate estimate of the model update for a much lower cost. In this paper, we apply statistical sampling with LSRTM to reduce its cost.

Theory

In LSRTM, a reflectivity model is sought to best fit the observed data with a linear Born modeling operator by minimizing the misfit

$$f(\boldsymbol{m}) = \frac{1}{2}\|\boldsymbol{Lm} - \boldsymbol{d}\|^2, \qquad (1)$$

where the L2 norm squared indicates summations over time, shots, and receivers. $\boldsymbol{d}$ is the recorded data; $\boldsymbol{L}$ is the Born model modeling operator based on a given velocity model; and $\boldsymbol{m}$ is the reflectivity model. Since the direct solver is prohibitively expensive, an iterative solver is usually adopted, such as a steepest descent method

$$\boldsymbol{m}^{(k+1)} = \boldsymbol{m}^{(k)} - \alpha[\boldsymbol{L}^{\boldsymbol{T}}(\boldsymbol{Lm}^{(k)} - \boldsymbol{d})]. \qquad (2)$$

We use a conjugate gradient method to minimize the misfit in equation (1).

With the constant-density acoustic wave equation, the Born modeling operator can be expressed as

$$\begin{cases} \frac{1}{v^2}\frac{\partial^2 P}{\partial t^2} - \nabla^2 P = S(t) \\ \frac{1}{v^2}\frac{\partial^2 Q}{\partial t^2} - \nabla^2 Q = mP \end{cases}, \qquad (3)$$

where P and Q are source and receiver side wavefields, v is the given velocity, and $S(t)$ represents a source function. Note that the source for the wavefield Q is the wavefield P scaled by the reflectivity model m. The Q wavefield is recorded at receiver locations to give a synthetic dataset for comparison against the observed data. The RTM operator $\boldsymbol{L}^{\boldsymbol{T}}$ in equation (2) is calculated as

$$\begin{cases} \frac{1}{v^2}\frac{\partial^2 P}{\partial t^2} - \nabla^2 P = S(t) \\ \frac{1}{v^2}\frac{\partial^2 \tilde{Q}}{\partial t^2} - \nabla^2 \tilde{Q} = d \\ m = \sum_t P \times \tilde{Q} \end{cases}, \qquad (4)$$

with a cross-correlation imaging condition. Note that during the numerical calculation of the equation 4, the source side wavefield P propagates forward in time but the receiver side wavefield $\tilde{Q}$ propagates backward in time. Solving the problem in equation (1) iteratively gradually improve the image quality over the iterations.

The conventional approach is to include all the shots (in a particular geographic region) of the survey in the sum. With statistical sampling algorithm, only a subset of the shots is included at every iteration to reduce the computational cost.

Examples

The LSRTM algorithm was tested with part of the 3D SEAM model (Day et al. 2009). The synthetic data were generated with narrow azimuth acquisition geometry. There were a total of 4800 shots distributed along 20 source lines with 400-m cross-line (Y) spacing and 100-m in-line (X) spacing. Each shot was recorded with a towed streamer array consisting of 11 cables with 200-m spacing and each cable had101 receivers with an interval of 100 m. The synthetic data were calculated with variable-density isotropic wave equation and the true velocity and density model shown in Figure 1. A Ricker wavelet with 10 Hz maximum frequency was used as the source wavelet.

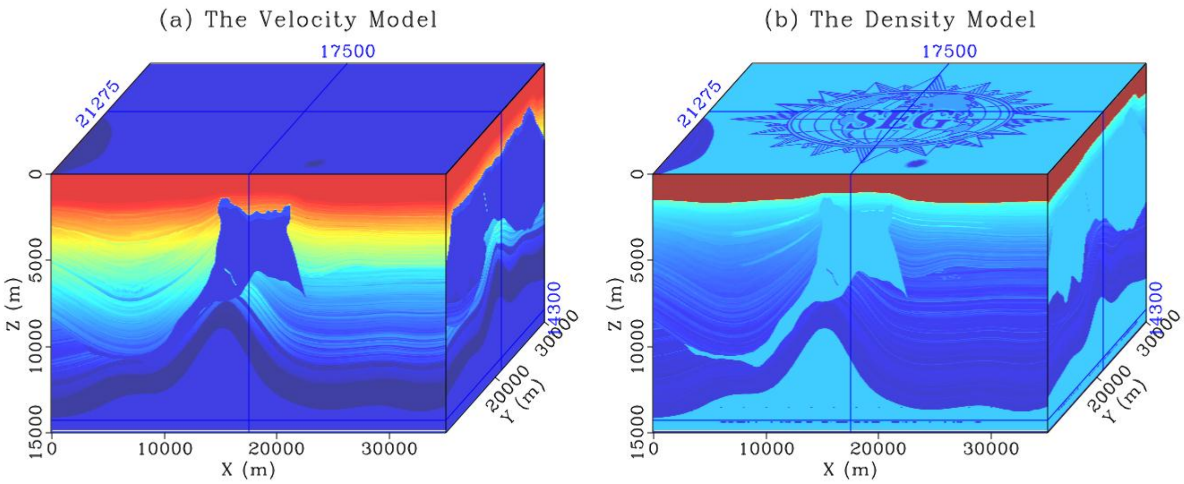

***Figure 1**: The SEAM velocity model (a) and density model (b) were truncated to the portion covering the SEG logo in the density model.*

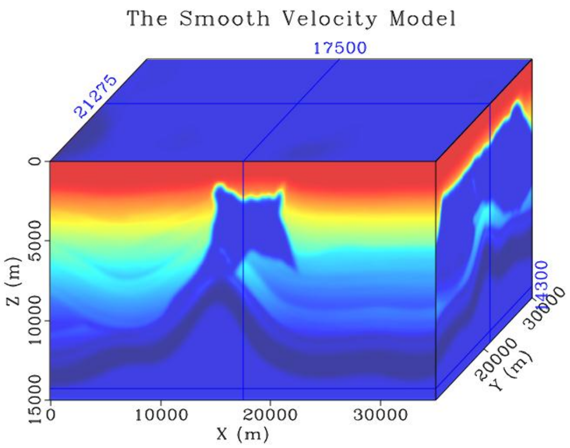

***Figure 2**: The smooth velocity model for migration obtained by 3D Gaussian smoothing of the true slowness model with a 500-m window.*

The data were migrated with the smooth velocity model in Figure 2 and constant density; an in-line section and a depth slice of the RTM image are shown in Figures 3a and 3b, with the same slices as Figure 1. Figures 3c and 3d shows the LRTM image after 10 iterations. In the in-line section, the overall amplitudes are balanced and the illumination of the subsalt part is improved. In the shallow part, the resolution of the horizontal reflectors is enhanced. Figures 4a and 4b plot the zoom views of the subsalt sections, where the LSRTM image delineates the syncline better than the RTM image does. In the depth slices, the SEG logo is resolved better in the LSRTM image with more balanced amplitudes and better resolution. If we assume each iteration of LSRTM needs twice the cost of RTM, the cost of LSRTM image is 20 times that of RTM.

To test the feasibility of the statistical sampling algorithm, the same data are migrated again with LSRTM, but at each iteration only about 8% of the shots are used. The shots are selected randomly

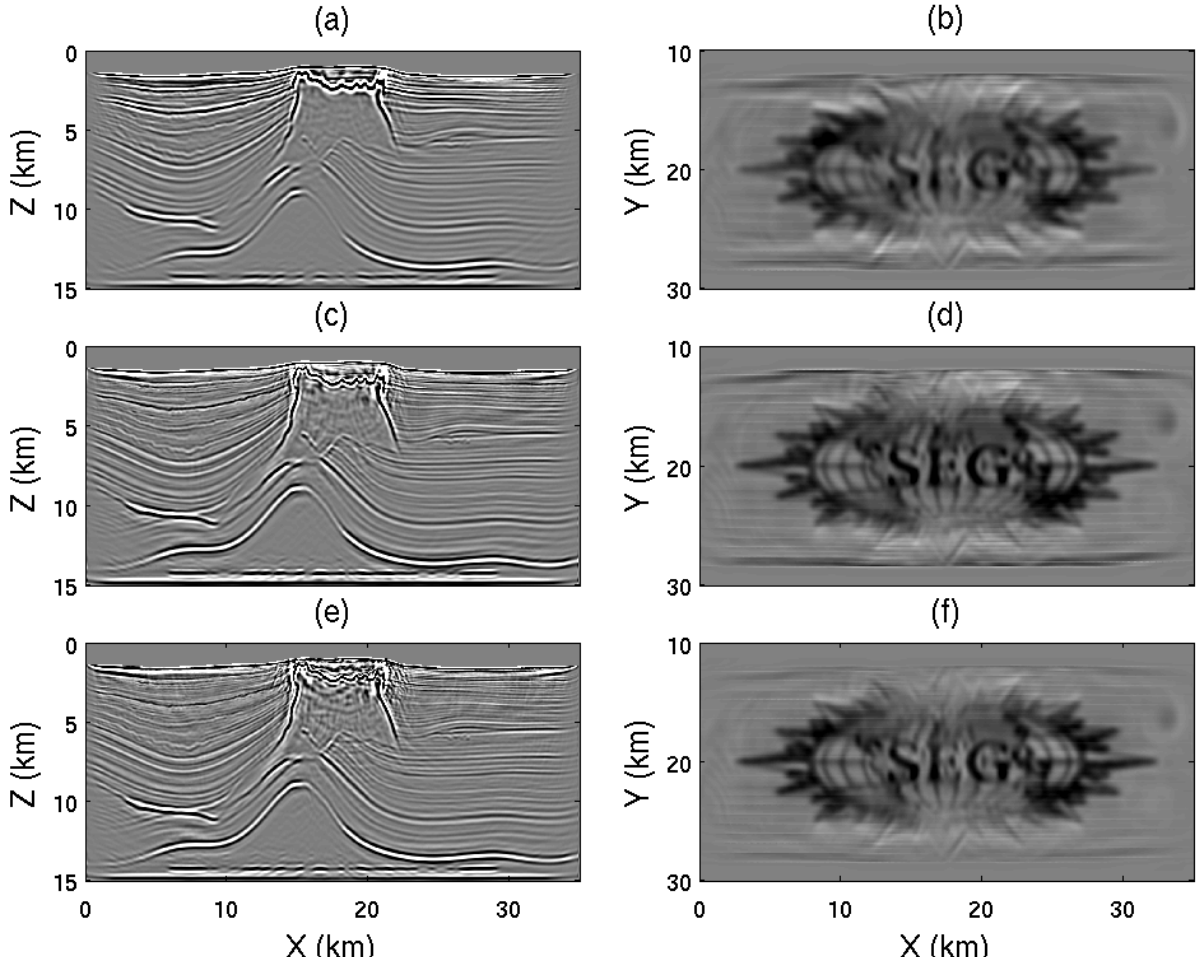

***Figure 3**: The in-line and depth sections of RTM and LSRTM images: (a) the in-line section (y=21.3km) of the RTM image; (b) the depth slice (z=14.3km) of the RTM image. Panels (c) and (d) show the same slices of LSRTM image after 10 iterations and panels (e) and (f) show the same slices of LSRTM image with statistical sampling after 15 iterations.*

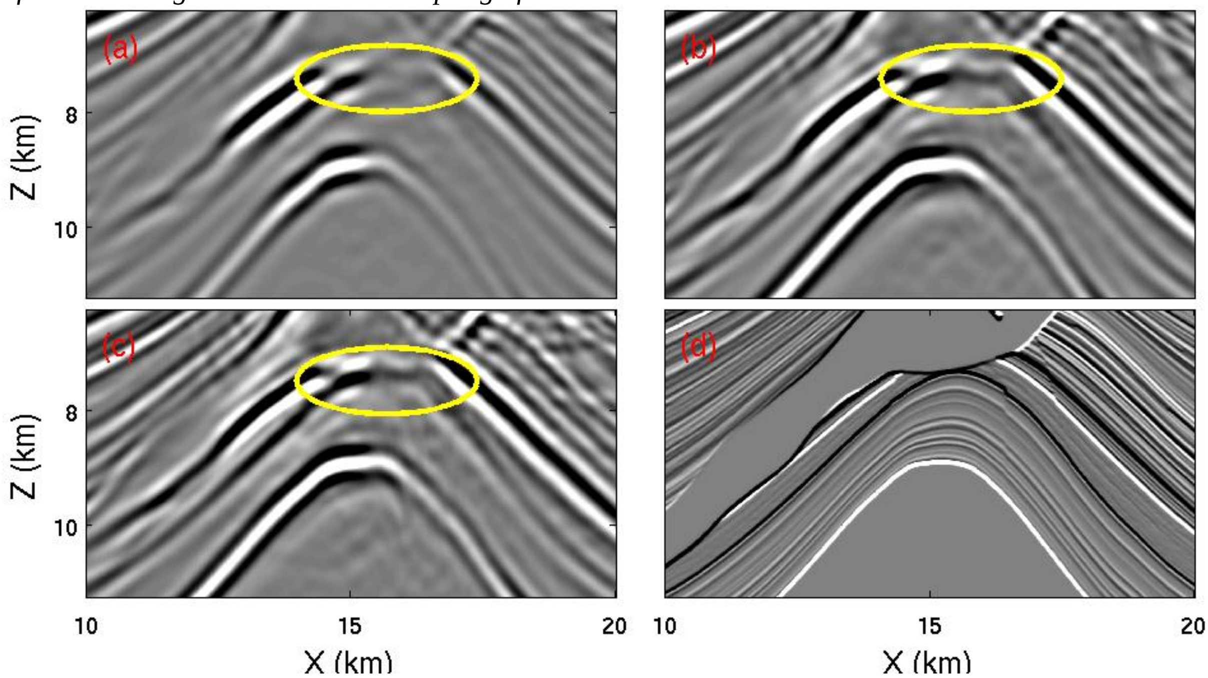

***Figure 4**: Zoom views of the subsalt sections in Figures 3a, 3c and 3e.The reflector in the yellow ellipse is better resolved in the LSRTM images (panels (b) and (c)). Panel (d) shows the true reflectivity.*

and changed at every iteration. Two slices of the image after 15 iterations are shown in Figures 3e and 3f, which are similar to Figures 3c and 3d, but their cost is only about 2.4 times that of RTM. The zoom view in Figure 4c also shows similar quality to Figure 4b. Figure 4d shows the true reflectivity as the benchmark.

Conclusions

LSRTM can improve the image quality by taking into account limited acquisition aperture, source wavelet, geometric spreading, etc. However, its application is limited due to high computational cost. In this paper, we propose to combine LSRTM with statistical sampling to reduce the computational cost. The proposed method is tested with a dataset of 3D SEAM model. With 2.4 times the cost of RTM, LSRTM with statistical sampling can produce an image showing similar quality improvements of full LSRTM, compared to a conventional RTM image. With better efficiency, the LSRTM with statistical sampling can be applied to realistic 3D dataset to produce better images over RTM.

Acknowledgements

We thank the management of Schlumberger for permission to publish this paper.

References

Boonyasiriwat C. and Schuster G. T. [2010] 3D multisource full-waveform inverions using quasi-Monte Carlo phase encoding. SEG Technical Program Expanded Abstracts, 1044-1049.

Dai, W., Fowler P., and Schuster G. T. [2012] Multisource least-squares reverse time migration. Geophysical Prospecting, **60**, 681–695.

Dai, W. and Schuster G. T. [2013] Plane-wave least-squares reverse-time migration. Geophysics, **78**(6), S165–S177.

Dai, W., Huang Y., and Schuster G. T. [2013] Least-squares reverse time migration of marine data with frequency-selection encoding. Geophysics, **78**(4), S233–S242.

Day, R., Naylor E., Stefani J., and Stoughton D. [2009] Construction & Composition of the SEAM Phase I Model: Presented at the 79th Annual International Meeting, SEG.

Krebs, J. R., Anderson J. E., Hinkley D., Neelamani R., Lee S., Baumstein A., and Lacasse M.-D. [2009] Fast full-wavefield seismic inversion using encoded sources. Geophysics, **74**(6), WCC177–WCC188.

Romero, L. A., Ghiglia D. C., Ober C. C., and Morton S. A. [2000] Phase encoding of shot records in prestack migration. Geophysics, **65**, 426–436.

Schuster, G. T., Wang X., Huang Y., Dai W., and Boonyasiriwat C. [2011] Theory of multisource crosstalk reduction by phase-encoded statics. Geophysical Journal International, 184, 1289–1303.

Van Leeuwen, T. and Herrmann F. [2012] Fast waveform inversion without source-encoding. Geophysical Prospecting, **61**, 10-19.

Zhang, Y., Duan, L., and Xie, Y. [2013] A stable and practical implementation of least-squares reverse time migration. SEG Technical Program Expanded Abstracts, 3716-3720.